EC38044 Continuous-Time Macroeconomics and Finance

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1 Overview

Course Objectives: Continuous-time methods are increasingly been applied in Macroeconomics and Finance due to its elegance, its relative easiness to derive analytical results, and its ability to take advantage of existing computational methods in Applied Mathematics. Nevertheless, technical hurdles prevents its usage from a wider audience. The course will thus be partly methodological, and partly about essence, with the objective of being accessible to graduate students in Economics and Finance.

The first part of the course deals with heterogeneous-agent models in a continuous-time setting, and the second part martingale theory of asset pricing with general equilibrium applications. Mathematical background and computational methods are introduced along the way.

Organization: A preliminary schedule includes 5 lectures, each with 2.5 hours. The schedule is available at
`https://cloud.timeedit.net/su/web/stud1/ri15c12Q054Z0ZQ6356357a0yn066W73h1Z69Q0Q0576QZy1xxuYvQnw.html`

Assessment: There will be problem sets to be completed individually at home, which include both theoretical derivations and computational exercises. Evaluations will be based on the problem sets. Active participation in classes generates bonus points, including thinking actively about potential research topics related to the course.

2 Topics

Lecture 1: Brownian Motion and Itô Calculus

> This lecture introduces key concepts of Stochastic Calculus: Brownian motion, stochastic differential equations (SDE) and Itô Lemma. Random-walk representation will be introduced to form an intuitive understanding of the SDE.
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> References:
> More accessible: Lawler (2023), Stokey (2008) and Dixit (1993).
> More technical: Karatzas and Shreve (2014), Evans (2010).

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Lecture 2: Heterogeneous-Agent Model à la Aiyagari (1994).

This lecture applies the tools of the previous lecture in a Aiyagari (1994)-type of model, which is reformulated in a continuous-time setting by Achdou et al. (2022). We derive the Hamilton-Jacobi-Bellman (HJB) equation and the Kolmogorov Forward (KF) equation, as well as numerical algorithms to solve them.

References:
Core: Achdou et al. (2022), Aiyagari (1994)
Related: Gabaix (2009), Gabaix et al. (2016), Allaire (2007), Barles and Souganidis (1991).

Lecture 3: Arbitrage and Martingale Theory

We introduce the concept of non-arbitrage and its relationship with the existence of martingale measure, i.e. the first fundamental theorem of asset pricing. We also talk about the uniqueness of martingale measure under complete market, i.e. second fundamental theorem of asset pricing. Relevant probabilistic knowledge around the concept of martingale, including filtration, Radon-Nikodym derivative etc, will be introduced along the way.

Reference:
Core: Björk (2020)
Related: Black and Scholes (1973), Shreve et al. (2004), Shreve (2005), Cochrane (2009).

Lecture 4: Portfolio Choice and Dynamic Equilibrium in Complete Markets

We apply the concepts of the previous lecture in a complete market model à la Merton (1971). Two approaches will be used to solve the model: dynamic programming (i.e. HJB), and martingale approach. An equilibrium model will also be considered to pin down the price of basic assets.

Reference:
Core: Merton (1971), Duffie (2010), Karatzas et al. (1998)
Related: Dumas and Luciano (2017), Merton (1973), Cox et al. (1985)

Lecture 5: An Incomplete Market Model

The martingale measure is not unique under incomplete market and this lecture discusses how to choose the relevant martingale measure for portfolio optimization problem. We will also talk about an equilibrium model if time permits.

Reference:
Core: Björk (2020), Basak and Cuoco (1998)
Related: Brunnermeier et al. (2022)

References

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., and Moll, B. (2022). Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, 89(1):45–86.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Allaire, G. (2007). *Numerical analysis and optimization: an introduction to mathematical modelling and numerical simulation*. OUP Oxford.

Barles, G. and Souganidis, P. E. (1991). Convergence of approximation schemes for fully nonlinear second order equations. *Asymptotic analysis*, 4(3):271–283.

Basak, S. and Cuoco, D. (1998). An equilibrium model with restricted stock market participation. *The Review of Financial Studies*, 11(2):309–341.

Björk, T. (2020). *Arbitrage theory in continuous time*. Oxford university press, 4th edition.

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, 81(3):637–654.

Brunnermeier, M. K., Merkel, S., and Sannikov, Y. (2022). Lectures on macro, money, and finance: A heterogeneous-agent continuous-time approach. Preliminary and Incomplete Lecture Notes, Princeton University and Stanford University.

Cochrane, J. H. (2009). *Asset pricing: Revised edition*. Princeton university press.

Cox, J. C., Ingersoll Jr, J. E., and Ross, S. A. (1985). An intertemporal general equilibrium model of asset prices. *Econometrica: Journal of the Econometric Society*, pages 363–384.

Dixit, A. K. (1993). *The Art of Smooth Pasting*, volume 55. Taylor & Francis.

Duffie, D. (2010). *Dynamic asset pricing theory*. Princeton University Press.

Dumas, B. and Luciano, E. (2017). *The economics of continuous-time finance*. mIT Press.

Evans, L. C. (2010). *Partial Differential Equations*, volume 19. American Mathematical Soc.

Gabaix, X. (2009). Power laws in economics and finance. *Annu. Rev. Econ.*, 1(1):255–294.

Gabaix, X., Lasry, J.-M., Lions, P.-L., and Moll, B. (2016). The dynamics of inequality. *Econometrica*, 84(6):2071–2111.

Karatzas, I. and Shreve, S. (2014). *Brownian motion and stochastic calculus*, volume 113. springer.

Karatzas, I., Shreve, S. E., Karatzas, I., and Shreve, S. E. (1998). *Methods of mathematical finance*, volume 39. Springer.

Lawler, G. F. (2023). Stochastic calculus: An introduction with applications. `https://www.math.uchicago.edu/~lawler/finbook.pdf`.

Merton, R. C. (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, 3(4):373–413.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, pages 867–887.

Shreve, S. (2005). *Stochastic calculus for finance I: the binomial asset pricing model*. Springer Science & Business Media.

Shreve, S. E. et al. (2004). *Stochastic calculus for finance II: Continuous-time models*, volume 11. Springer.

Stokey, N. L. (2008). *The Economics of Inaction: Stochastic Control models with fixed costs*. Princeton University Press.