Improved numerical stabilisation of grounding line problems

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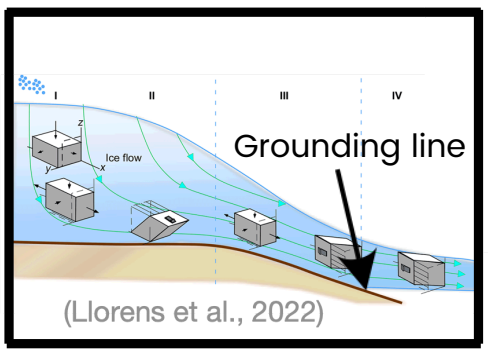
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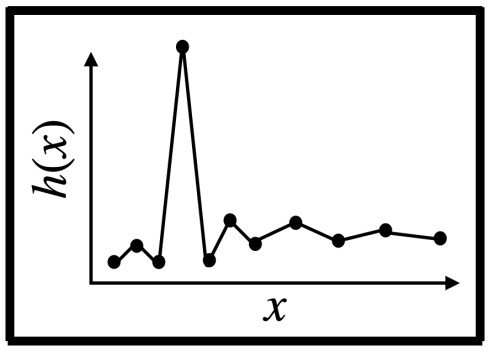
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- Increase in stable time step from 1 yr to 50 yr
- Correction of the force balance mismatch at the grounding line

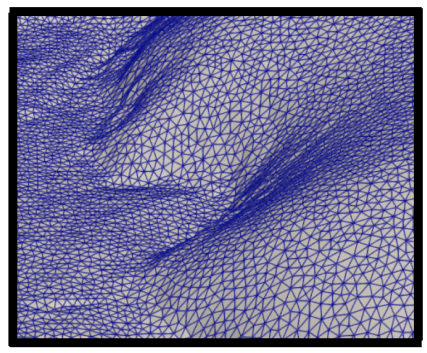
1. Modelling challenges



Model representation of complex physics

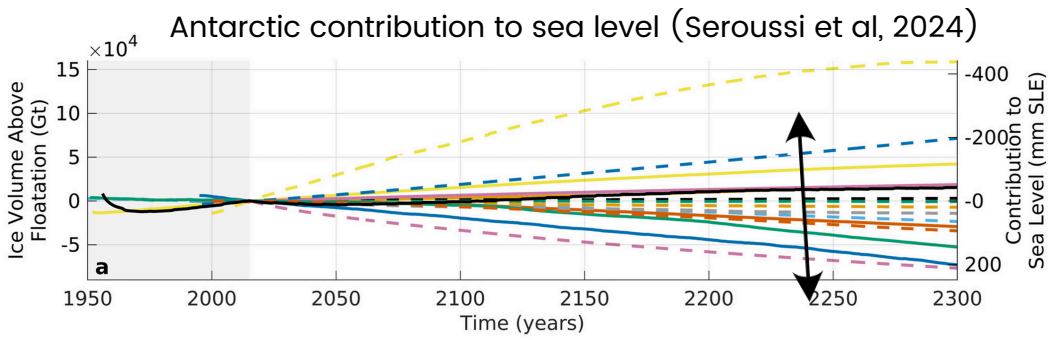


Issues with numerical instability



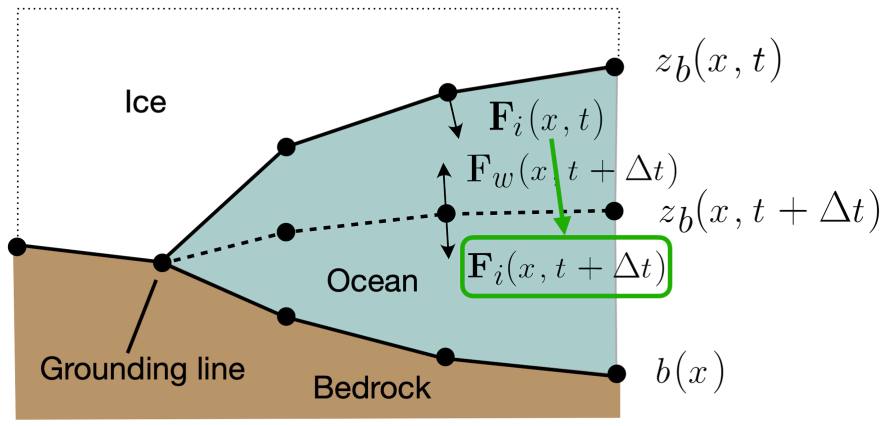
Mesh resolution restrictions

2. Ice-sheet modelling state-of-the-art



- Ice-flow models predict differing Antarctic sea-level contribution
- Continental ice-sheet model physics is restricted due to time-step constraints
- No full Stokes model used in this study

3. Grounding line stabilisation



- FSSA: Predicted ice load at the next time step
- Sea spring: Predicted buoyancy force at next time step (Durand et al., 2009)

7. Numerical stabilisation

- Reynold's Transport Theorem used in Free-Surface Stabilisation Scheme (Kaus et al., 2010; Löfgren et al., 2022):

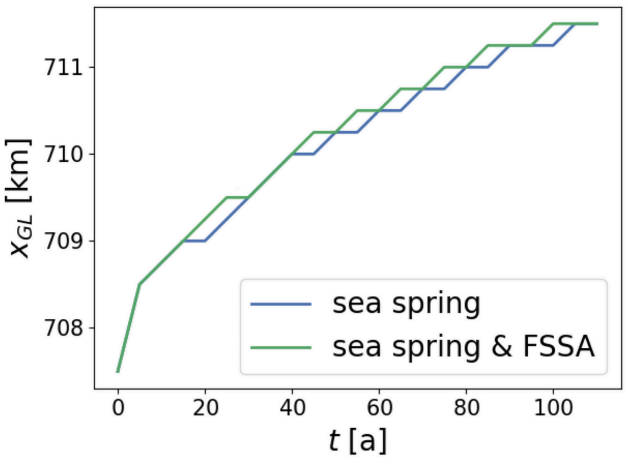
$$\frac{\partial}{\partial t}\int_{\Omega(t)} f\,dV = \int_{\Omega(t)} \frac{\partial}{\partial t} f\,dV + \int_{\partial\Omega(t)} (\mathbf{u}\cdot\mathbf{n}) f\,dV$$

- Adapted to ice-flow modelling as boundary conditions in the weak form of the Stokes equations:

$$\int_{\Omega(t+\Delta t)} \rho\mathbf{g}\cdot\mathbf{v}\,dV \approx \int_{\Omega(t)} \rho\mathbf{g}\cdot\mathbf{v}\,dV + \theta\Delta t \int_{\partial\Omega(t)} (\mathbf{u}\cdot\mathbf{n} + a_s)(\rho\mathbf{g}\cdot\mathbf{v})\,dV$$

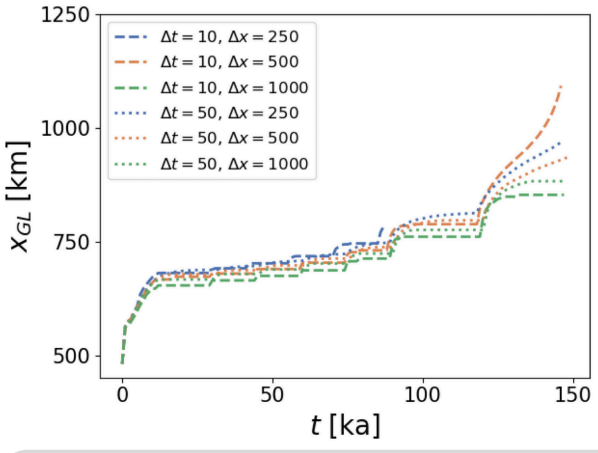
$$\int_{\Omega(t+\Delta t)} \rho\mathbf{g}\cdot\mathbf{v}\,dV \approx \int_{\Omega(t)} \rho\mathbf{g}\cdot\mathbf{v}\,dV + \theta\Delta t \int_{\partial\Omega(t)} (\mathbf{u}\cdot\mathbf{n} + a_b)(\rho\mathbf{g}\cdot\mathbf{v})\,dV$$

8. Force balance correction



- Force balance mismatch causes grounding line migration delay when only using sea spring

9. Increased stable time step



- Stable time steps of 50 years possible in MISMIP benchmark simulations!
- Significant improvement compared to current time step sizes of ~0.1 years

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