PHD TOPICS: SPECTRAL THEORY OF PARTIAL DIFFERENTIAL EQUATIONS

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Spectral theory of differential operators has a long history in mathematics and physics, and eigenvalues of the Laplacian and other elliptic operators appear in a variety of physical phenomena, from the frequencies of a vibrating string or membrane to the energies of quantum systems. Lord Raighley's book *The Theory of Sound* from 1877 can be considered as the starting point of modern spectral theory, and the field has taken an enormous development since then, being stimulated greatly by the formation of quantum mechanics during the first decades of the twentieth century. Nowadays spectral theory is a huge and immensely active field in mathematics and physics.

PhD topics under my supervision may be connected to the interplay between the spectra of differential operators and the geometry of the underlying space; this area of research is called spectral geometry. Possible topics could for instance be:

- Critical points of eigenfunctions, and their global maxima and minima, for certain elliptic differential operators. A prototypical problem is the famous *hot spots conjecture* on the large-time behaviour of the hottest and coldest spots in an insulated physical body.
- Nodal properties of eigenfunctions, where the zero lines (more generally zero hypersurfaces) of eigenfunctions are studied. Their patterns have large physical significance and appeared in Chladni's plate experiments already more than 200 years ago. A famous open problem is Payne's nodal line conjecture stating that the nodal set of the second Dirichlet Laplacian eigenfunction should touch the boundary.
- Shape optimization problems for partial differential operators such as the Laplacian, i.e. questions similar to the subject of the famous Faber-Krahn inequality: within the class of all Euclidean domains or Riemannian manifolds having a geometric quantity fixed (e.g. fixed volume or surface area), which shape optimizes the first (or k-th) eigenvalue?

Projects under my supervision require a good knowledge in functional analysis and ordinary or partial differential equations and a general interest in spectral theory, as well as the willingness to carry out significant research within a lively area of mathematics.